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Kant on the Acquisition of Geometrical Concepts
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Keywords:	Kant; concepts; geometry; construction; phenomenology
Abstract:	<p>It is often maintained that one insight of Kant's Critical philosophy is its recognition of the need to distinguish accounts of knowledge acquisition from knowledge justification. In particular it is claimed that Kant held that the detailing of a concept's acquisition conditions is insufficient to determine its legitimacy. I argue that this is not the case at least with regard to geometrical concepts. Considered in the light of his pre-Critical writings on the mathematical method, construction in the Critique can be seen to be a form of concept acquisition, one that is related to the modal phenomenology of geometrical judgment</p>

Kant on the Acquisition of Geometrical Concepts

1. Construction and Concept Acquisition

Kant thought of concepts as falling into three broad classes. As well as empirical concepts, we are in possession of two types of a priori concepts.¹ In the first case, there are ‘pure sensible’ concepts, i.e. *mathematical concepts* (e.g. see A140-1/B180); secondly, there are what I’ll refer to as *categorical concepts* (i.e. the Categories), concepts that are analogues of traditional metaphysical concepts such as *<cause>*, *<substance>*, etc.² That a concept is a priori does not entail that it is non-acquired, however. Kant repeatedly states that *all* concepts, a priori concepts included, are acquired.³ For Kant, the acquisition procedure for empirical concepts is conceived along the lines of a Lockean abstraction procedure.⁴ Mathematical concepts, I will argue, are acquired through a special process of definition involving a procedure Kant calls ‘construction.’⁵

In this paper I present an account of Kant’s account of geometrical construction, understood as a concept acquisition procedure.⁶ Construction has not usually been considered as a concept acquisition procedure and one possible reason for this is that Kant’s well-known example of construction, the proof procedure of Euclid’s I.32 in the *Discipline of Pure Reason*, seems to begin from a scenario where two inquirers, a philosopher and a mathematician, are *already* in possession of a concept (in this case, the concept *<triangle>*). It therefore seems that Kant is indicating that the construction procedure is a process of validating the already acquired concept by way of Euclidean proof. As such, construction is it seems better characterized as a justification procedure

for an already-possessed concept rather than a concept acquisition procedure for that concept.

Another worry regarding treating construction as an acquisition procedure concerns Kant's general Critical approach to epistemic justification. Part of Kant's 'normative turn', it is often supposed, is a general opposition to the justification of a concept's conditions of use by providing a full account of that concept's possession conditions. The *quaestio juris* with regard to a concept's employment in judgment is supposed to have been seen by Kant as not sufficiently secured by the successful answering of a *quaestio facti* with regard to the conditions under which that concept has come to be acquired (A84-5/B116-7). If construction is indeed a process of concept acquisition then – since it seems that Kant certainly does regard construction as a justification procedure – it would entail that a sufficient account of the acquisition conditions for an a priori concept can indeed suffice to provide justification for that concept's use. On such an interpretation Kant's account of construction would offer a significant qualification, if not a reformulation, of his perceived normative ambitions in the first *Critique*.

Despite these concerns, I would claim that a case can be made for interpreting construction as a concept acquisition procedure. The account presupposes a broader methodological perspective on Kant's epistemological project in the first *Critique* and in particular the nature of Kant's 'how possible?' question. In asking how synthetic a priori propositions are possible, Kant is asking how it is that we have come to possess the knowledge that we in fact possess.⁷ This question should be explored as an expression of a more specific question, namely, how is it that we have come to possess the concepts that figure in those knowledge-generating and knowledge-preserving judgments? Pursuing Kant's epistemological project along these lines can, I claim, provide a fruitful reading of his account of mathematical knowledge.⁸

This is the central feature of Kant's famous discussion at A716-7/B744-5 of the geometer's proof of proposition I.32 of Euclid's *Elements*, that the internal angles of all triangles necessarily equal the sum of two right angles. Specifically, Kant is attempting to show that the full-blooded mathematical concept of triangularity could itself only have been acquired through a proof involving intuition. In this sense the diagrammatic proof procedure itself constitutes the acquisition conditions for that concept. Only through such reasoning, Kant claims, could the epistemic access to necessarily true propositions be secured. Combining the two claims, I argue that Kant reverses the explanatory order of the Leibnizian account of the relation between concept possession and knowledge. We do not satisfy the possession conditions for a geometrical concept and then use that possessed concept to secure a priori knowledge, but rather we meet the possession conditions for that concept by way of securing a priori knowledge about that concept's extension.

The standard reading of this example is that it is directed to showing the necessity of intuitions for the acquisition of synthetic a priori knowledge. In this case the requirement is that of the performance of a proof procedure upon a spatial particular in the form of a geometrical diagram. Kitcher's comment captures what can appear to be Kant's methodology here:

We are supposed to gain a priori knowledge of the elementary properties of triangles by using our grasp on the concept of triangle to construct a mental picture of a triangle and by inspecting this picture with the mind's eye. (1980, 8)

I claim that this reading gets Kant's intentions the wrong way around. The example of I.32 is in fact supposed to show that we must represent an empirical or mental image of a triangle in order to acquire the concept of a triangle. The example is primarily directed

towards showing how the geometer acquires the full, developed mathematical concept <triangle>. Central to the account presented here is that it is only through the deployment of concrete representations of particular triangles that the appropriate modal phenomenology involved in proof procedures can be generated. Furthermore, it is only via a procedure that generates the appropriate modal phenomenology that, Kant thinks, the a priori content of such concepts and the necessarily true judgments formed with them can be explained.

The aims and context of Kant's discussion of geometrical concept acquisition can only be appreciated against the Pre-Critical development of his views. The questions pursued in the *Critique* concern Kant's submission to the Berlin Academy's Prize Essay competition, the *Inquiry Concerning the Distinctness of the Principles of Natural Theology and Morality*,⁹ published in 1764 as the runner-up submission. The question set for the competition was whether metaphysical propositions could be proven and known with a certainty comparable to that of the propositions of geometry. The *Inquiry* expressed Kant's growing disillusion at the time for the prospects for metaphysics, though many of the claims from the *Inquiry* survived directly into the 'dogmatic use' section in the *Critique* nearly two decades later, when Kant had regained his optimism. I'll first outline the claims in the *Inquiry* that found their way into the Critical system. Secondly, I will outline the new model of mathematical cognition presented in the 'dogmatic use' section in the *Critique*. Thirdly, I will present the alternative reading of the example of the Euclidean proof of proposition I.32. Finally, I will consider some of the implications of the reading suggested here.

2. Geometrical Concept Acquisition in the *Inquiry*

By 1763, when Kant was composing the *Inquiry*, he had already come to hold that all previous metaphysicians had laboured under a misapprehension, namely that they could imitate the methodology of mathematics. That this was in fact impossible would have been clear to them had they paid sufficient attention to how mathematics is actually practiced, and specifically to the crucial issue of the conditions under which we come to possess mathematical concepts.¹⁰ In mathematics we acquire the relevant concepts through a self-conscious, voluntary and creative act of *defining* them, by bringing together already possessed sub-concepts into a synthetic whole:

There are two ways in which one can arrive at a general concept: either by the arbitrary combination of concepts, or by separating out that cognition which has been rendered distinct by means of analysis. Mathematics only ever draws up its definitions in the first way. For example, think arbitrarily of four straight lines bounding a plane surface so that the opposite sides are not parallel to each other. Let this figure be called a *trapezium*. The concept which I am defining is not given prior to the definition itself; on the contrary, it only comes into existence as a result of that definition. Whatever the concept of a cone may ordinarily signify, in mathematics the concept is the product of the arbitrary representation of a right-angled triangle which is rotated on one of its sides. In this and in all other cases the definition obviously comes into being as a result of *synthesis*. (*Inquiry*, 2:276)¹¹

Kant's account depends on a distinction between concepts that are created and those that are 'given.'¹² That a concept is given for Kant does not entail that it is non-acquired, but rather that it has been acquired on some non-arbitrary grounds. One possibility is that the non-arbitrary grounds are that certain concepts have a pragmatic indispensability to the course of ordinary experience.¹³ Thus there are some concepts that, whatever the particular manner of their acquisition, are routinely acquired for the purposes of the

minimal representation of the world by ordinary agents. This is suggested by Kant's use of *<time>* as an example of a given concept – citing Augustine, Kant characterizes it as a concept that we all take ourselves to possess though one of which we also all lack a clear understanding (*Inquiry*, 2: 283-4). The concept is presumably pragmatically indispensable: it and its cognates are required for an enormous range of simple communicative acts, for example; yet this indispensability does not entail that we have a sound understanding of just what time is. It is this circumstance in which we find ourselves – possessing concepts of which we all claim some minimal mastery but of which we lack a full understanding – that prompts inquiry itself. Since metaphysical concepts are given, a definition represents not so much the starting point but rather the end point for inquiry.¹⁴

By contrast, a voluntarily created concept is marked out by the 'arbitrariness' of this act of creation. An arbitrary combination should not be taken to entail that the content of the concept formed is in any way contingent – it merely marks the fact that the concept's possession is contingent, since it has taken place through a self-conscious decision to form that concept, presumably without being prompted by the pragmatic needs that stimulate the acquisition of given concepts. In metaphysics we proceed towards a definition through decomposition of a concept into its fundamental sub-concepts, and this presents the most important point of contrast with the method of geometry – 'geometers acquire their concepts by means of *synthesis*, whereas philosophers can only acquire their concepts by means of *analysis* – and that completely changes the method of thought.' (*Inquiry*, 2: 289)¹⁵

Although it can occasionally appear that the mathematician is putting forward analytic definitions, mathematics always functions in the synthetic definitional manner – to think otherwise 'is always a mistake' (*Inquiry*, 2: 277). If it appears that one is making inferences that follow directly from the meaning of a given concept, it will always turn out that 'in the end nothing is actually inferred from such definitions, or, at any rate, the

immediate inferences which he draws ultimately constitute the mathematical definition itself (*Inquiry*, 2: 277). Thus in the *Inquiry* Kant holds that not only are the possession conditions for mathematical concepts the results of an activity of synthesis, but also that those possession conditions can in fact be constituted by the *inferences* that the mathematician performs. If it appears that a mathematician is making an analytic definition by decomposing concepts, it can turn out that this mistaken impression is caused by the fact that just what it is to grasp a concept's content is just to be able to make certain rational rule-governed procedures over particular figures. Kant's own example is instructive here: what it is to grasp the concept <cone> is constituted by the subject's grasp of an operation upon a geometrical figure, that of rotating the figure of a triangle along one of its sides. Moreover, there is a kind of reciprocal relation here between two capacities – the capacity to grasp a concept's content on the one hand and the capacity to represent a token of the type expressed by that concept on the other. What is required for a subject to represent a token cone is just that the subject can understand an operational rule regarding the rotation a triangle, the latter which just constitutes the content of the concept <cone>. Conversely, what is required for a subject to grasp the concept <cone> is just that the subject can perform the operation required for representing a token of that type.

Kant's account of geometrical knowledge hinges upon the epistemic role of 'individual signs' (*Inquiry*, 2: 279). Individual signs are representations whose explicit intentional content is the presentation of a particular (they present the universal '*in concreto*', as Kant puts it). This is in contrast to natural language, which 'represent[s] the universal *in abstracto*' (*Inquiry*, 2: 279), i.e. words are representational vehicles that can purport to capture the generality of their content, e.g. as with 'triangularity'.¹⁶ The signs deployed in metaphysics are invariably signs *in abstracto* in the form of natural language. Ironically though, the fact that natural language consists of such signs actually constitutes

an impediment for its task of expressing universal claims. In Kant's view, signs *in abstracto* are clearly inferior to signs *in concreto* with regard to their power to express the universals with which they are concerned. For example, a sign *in concreto* in geometry might be a drawing of a triangle, which provides a concrete instantiation of the universal of triangularity. In the *Inquiry* Kant seems to be as of yet unperturbed by the generality problem in geometry, i.e. concerning just how the use of a particular example might suffice as an exemplar on the basis of which general claims might be made.¹⁷ Rather, at this point he seems to take it for granted as a remarkable fact that mathematics, unlike metaphysics, just *is* a practice whereby 'to discover the properties of all circles, one circle is drawn' (*Inquiry*, 2:278). There is therefore a link made in the pre-Critical period between the possible representation of tokens of a particular type and the possibility of grasping necessary truths about the relevant type.

For this reason, Kant thinks, 'nothing has been more damaging to philosophy than mathematics, and in particular the *imitation* of its method in contexts where it cannot possibly be employed.' (*Inquiry*, 2: 283) Yet this conclusion obscures the crucial insight that Kant brings from the *Inquiry* to the *Critique of Pure Reason*. In the Pre-Critical period, Kant had already held that individual signs are the indispensable means for grasping necessary truths about mathematical universals; according to the Critical model of cognition, *all* grasping of synthetic necessary truth must occur via the presentation of particulars through intuition. The *Inquiry* marks a crucial step in Kant's rejection of the rationalist model of discursive cognition, considered as knowledge gained through discursive representation alone through analysis. It is in the *Inquiry* that he first recognizes the importance of resisting the thought that the appropriate manner of expressing the generality of necessary truths through the exclusive use of signs *in abstracto*, such as the expression of concepts in natural language.¹⁸

To summarize, Kant was by 1763 already committed to some particular claims regarding the status of geometrical concepts. Firstly, he maintained that the security of propositions in geometry was grounded upon the security of geometrical concepts. Secondly, he maintained that the security of geometrical concepts was grounded by their having a valid origin, in the literal sense of having secure acquisition conditions. Thirdly, he maintained that the security of those acquisition conditions was achieved by virtue of the acquisition conditions for the specific intensional content essentially involving epistemic acquaintance with token representatives of the relevant extension of the concept acquired. Fourthly, he maintained that in some cases the intension of the relevant concept could itself be constituted by non-syllogistic inferences. This is to say that there are some rational rule-governed representational operations – such as rules for possible triangle-rotation – performed just for the purposes of representing tokens of the relevant type. This already presents a radical difference from the nature of empirical concept acquisition, whereby the achievement of representing tokens of a type is explanatorily prior to the achievement of acquiring the concept that is abstracted from those representations. On Kant's account of geometrical concept acquisition, there can be no explanatory priority between the representation of a token of a type and the acquisition of the concept that expresses that type.

3. Intuition and Construction

By the time Kant presented his account of mathematical cognition in the first *Critique*, nearly two decades later, his view of knowledge had changed radically. The change could be expressed by contrasting it with the rejected rationalist model of knowledge. The achievement of veridical representation for the rationalist is secured

through discursive representation alone, through pure acts of thinking. The transition from the state of lacking knowledge to the state of possessing it is to be understood as the transition from a state of indistinct and obscure thought to clear and distinct thought. The confusion of obscure thought is due to the impurities of sensory representations infecting the capacity of thought itself. Thus the achievement of knowledge is one that would present no challenge if our discursive capacity were allowed to perform its task unimpeded, since the outputs of the human discursive capacity are both necessary and sufficient for knowledge acquisition.

Kant's picture of the mind is, as with most early modern philosophers, considered in terms of certain representational capacities. As well as our discursive capacities, such as the understanding and reason, Kant claims we also possess a range of non-discursive representational capacities. Amongst those non-discursive representational capacities are the familiar ones of the sensory modalities, memory and imagination. The *Critique* though marks the recognition of the requirement of further, distinct type of non-discursive representational capacity, that of *intuition*.¹⁹ In a broader sense Kant's project is motivated by the denial of the rationalist picture and by the thought that the necessary co-deployment of both discursive and non-discursive capacities can – on occasion – be the jointly sufficient conditions for knowledge acquisition.

Although our intuitional capacity makes possible the receipt of sensory representational content, intuition also contributes its own representational content.²⁰ For any given perceptual experience, that experience will contain representational content that is provided solely by the sensory modalities, but it will also contain content that, though capable of being expressed through several sensory modalities,²¹ is not itself generated by the senses (either individually or in complex combination). Kant claims that for any given perception of an object, the *spatiotemporal* content of that representation

does not come within the sensory content that constitutes the ‘matter’ of perceptual experience (A20/B34). Yet given that we do in fact represent spatiotemporal features of objects, and assuming that the products of senses exhaust the non-contributed portion of human beings’ representational content, it must be the case that the spatiotemporal content within our perceptual representations is *contributed* content (B1-2). For Kant our intuitional capacity can also be activated either with regard to particular sensory experiences or in the imagination (A713/B741). When this activity of the interaction of our imaginal and intuitional capacities occurs, the representations produced in imaginal space are ‘pure’ (B3).²²

In the ‘dogmatic use’ section of the *Critique* Kant retains many of the previously expressed commitments, most notably his claim that there is an essential difference between the methods of metaphysics and mathematics. By the time of the Critical system, Kant was still of the opinion that the superiority of mathematics was related to its ability to ‘go back to the sources of its concepts’. This difference is now expressed in terms of a special procedure that is particular to mathematics, that of *construction*. In explaining why definitions can only be provided in mathematics, he expresses many of the previous claims from the *Inquiry*:

Thus there remain no other concepts that are fit for being defined than those containing an arbitrary synthesis which can be constructed *a priori*; and thus only mathematics has definitions. For the object that it thinks it also exhibits *a priori* in intuition, and this can surely contain neither more nor less than the concept, since through the explanation of the concept the object is originally given, i.e., without the explanation being derived from anywhere else. (A729-30/B757-8)

Kant’s account of construction, I will argue, involves the amalgamation of several of the

key claims seen before: construction itself is nothing but the acquisition of concepts through acts of definition, where the latter is understood as manifested by inferential procedures performed upon concrete representations of particulars. Only through this procedure can certain objects even be thought, since those concrete representations are constitutive of the proof-procedures that must be performed even to acquire the developed concepts of mathematics. Whereas before the deployment of individual signs was a mere ‘aid to thought’, Kant now views the deployment of representations of particulars, in the form of intuitions, as necessary for the acquisition of the relevant conceptual content and the very capacity to think of the essential properties of the objects that fall under such concepts.

It is worth noting that the first *Critique* abounds with unambiguous statements to the effect that mathematical concepts are those that we create.²³ Kant couldn’t be more explicit as to the nature of mathematical definition – it is a concept-creation procedure:

[P]hilosophical definitions come about only as expositions of given concepts, but mathematical ones as constructions of concepts that are originally made, thus the former come about only analytically through analysis (the completeness of which is never apodictically certain) while the latter come about synthetically, and therefore **make** the concept itself, while the former only **explain** it. (A730/B758)

Furthermore, Kant is clear that the task of creating a mathematical concept can be identical to and coterminous with the task of representing a member of that concept’s extension. In the conclusion to the section on the Postulates of Empirical Thinking in General, Kant describes a mathematical postulate as follows:

Now a postulate in mathematics is the practical proposition that contains nothing

but the synthesis by which we first give to ourselves an object and generate its concept—e.g., to describe a circle with a given line from a given point on a plane – and such a proposition cannot be proved, because the procedure it requires is precisely that by which we generate the concept of such a figure. (A234/B287)

Similarly, Kant's well-known claim regarding synthesis in §24 of the B-Deduction seems to put a close connection between the very ability to think of something, which requires possession of the concept of that thing, with producing a representation of a token example of the thing in general, if only in the visual imagination. Kant claims both that

We cannot think of a line without **drawing** it in thought, we cannot think of a circle without **describing** it, we cannot represent the three dimensions of space at all without placing three lines perpendicular to each other at the same point... (B154)

The three examples used here are the concepts *<line>*, *<circle>* and *<three-dimensional space>*. In each case Kant claims that our capacity to possess this concept is dependent on our capacity to represent a referent of the concept. The capacity to think of a line is dependent on the capacity to draw a token line, the capacity to wield the concept *<circle>* is dependent on our capacity to present a circular object to consciousness. The third example, that of the capacity to grasp *<three-dimensional space>*, is made possible in a slightly different way. While our grasp of *<line>*, and *<circle>* is secured by the capacity to represent particular lines and circles, our ability to grasp *<three-dimensional space>* is secured by our ability to perform a certain geometrical operation upon a spatial manifold. Nevertheless, there is no doubt about the intended order of explanation here: it is not the case that we can put three lines perpendicular to each other at the same point *because* we have an antecedent conceptual grasp that space is three-dimensional; on the contrary,

it is because we can put three particular lines in that particular relation to each other that we are able to secure a conceptual grasp of the propositional content that space is three-dimensional. In all these cases, the *initial* grasp of these conceptual contents is directly tied to specific non-conceptual representational capacities to represent and manipulate spatial particulars in different ways.

In the *Prolegomena*, the distinction between a priori and a posteriori synthetic judgments is not drawn in terms of what allows for and what does not allow for successful concept-formation, but rather in terms of the status of the judgments that issue from the concept that is formed in each case:

...[J]ust as empirical intuition makes it possible for us, without difficulty, to amplify (synthetically in experience) the concept we form of an object of intuition through new predicates that are presented by intuition itself, so too will pure intuition do the same only with this difference: that in the latter case the synthetic judgment will be *a priori* certain and apodictic, but in the former only *a posteriori* and empirically certain, because the former contains only what is met with in contingent empirical intuition, while the latter contains what necessarily must be met with in pure intuition, since it is, as intuition *a priori*, inseparably bound with the concept *before all experience* or individual perception. (*Prolegomena*, 4: 281)

Here, Kant says that while empirical intuition allows for concept formation from which judgments regarding contingent truths can be made, pure intuition allows for the making of necessarily true judgments. Concepts formed from a posteriori individual perceptions afford judgments with a posteriori warrant; concepts formed from pure a priori intuition afford judgments with a priori warrant. The crucial innovation of the Critical model of cognition is to explain how we can access the *necessity* found in the judgments that can be formed by our use of such concepts.

This account of the a priori warrant for judgment relates to the account of geometrical construction presented in the Discipline. Here Kant claims that while neither empirical nor categorial concepts can be defined, mathematical concepts can be defined and this is just because mathematical concepts are created in voluntary acts of the understanding:

Since therefore neither empirical concepts nor concepts given *a priori* can be defined, there remain none but arbitrarily thought ones for which one can attempt this trick. In such a case I can always define my concept: for I must know what I wanted to think, since I deliberately made it up...(A729/B758).

Kant thus maintains the same position outlined in the *Inquiry*, namely that the possibility of definition is restricted to elective concepts, ones that have been ‘arbitrarily’ made up by the subject herself. It is also clear that Kant continues to maintain that the mathematician for just those same reasons secures epistemic certainty without difficulty. However, Kant identifies in the *Critique* a challenge that he did not raise for himself in the *Inquiry*. The challenge is how these two elements, the elective formation conditions of mathematical concepts, and the certainty of mathematical judgment, can be compatible. If, as he maintains, mathematical concepts are simply ‘arbitrarily’ created concepts, then how can mathematical knowledge be secured at all? Kant is well aware that a perennial source of error in the history of philosophy has been its practitioners’ disposition to indulge in the promiscuous formation of concepts without adequate warrant. Such ‘invented concepts’ can systematically generate false judgments, just because they have been formed arbitrarily and without concern to the very possibility of objects falling under them.²⁴ One might respond with the claim that elective concepts can have a positive truth-value, but that they are true only relative to an arbitrarily summoned-up

representation that is the ultimate referent of that concept. This though is to effectively characterize the concept as having a fictional status, since the objects that determine the concept's truth conditions are themselves arbitrarily created referents.

The challenge then is to show how mathematical concepts, as given by elective definitions, are not mere fictions as most voluntarily-formed concepts are. Kant's strategy for denying that elective definitions are error-theoretic is initially to concede that such acts of definitions are not sufficient for definition of 'a true object.' He contrasts the case of mathematical concept-formation with that of empirical concept-formation, using the example of *<chronometer>* for the latter, and claiming that

the object and its possibility are not given through this arbitrary concept; from the concept I do not even know whether it has an object, and my explanation could be better called a declaration (of my project) than a definition of an object.
(A729/B757)

With this empirical concept (one that combines the ideas of time-measurement and of a mechanical apparatus, say) I can grasp the content of the combined concept without having ever represented its referent. More pressingly, I can grasp the created content without knowing that it does have an actual or even possible object. For an arbitrarily created empirical concept, the conditions for grasping the intensional content and the conditions for representing its extension are distinct. As Kant concludes, the intensional content that is grasped is better thought of as a minimal set of directions for the pragmatic project of discovering the existence of the extension and its properties.

This feature of invented empirical concepts provides the contrast required for Kant's articulation of how an arbitrarily created concept *can* nevertheless be one whose possession conditions ensures epistemic certainty in its use. An arbitrarily created

concept can guarantee reference if production of an example of the extension is a necessary condition of the formation of the concept itself. This can occur if and only if the concept's intension itself is understood as a rule for the production of its referents. If a concept is one which can represent its extension in the same task of grasping its intension, then it is a concept that can be defined. This is what Kant claims does in fact occur on occasions of mathematical concept definition. In the passage previously quoted Kant claims that the only concepts 'that are fit for being defined are those containing an arbitrary synthesis which can be constructed *a priori*, and thus only mathematics has definitions.' (A729/B757) Kant is explicit here in linking the notions of definition, synthesis and construction. He says that a defined concept is one that 'contains' a rule for synthesis that itself makes possible the construction procedure. Kant's avoidance of the problem of invented concepts requires taking him as holding that the conditions under which the concept is formed are the very conditions under which a possible member of its extension is represented, and that these conditions are the conditions of construction.

4. Concept Acquisition and Content

Kant uses a different geometrical proof to express his point, and the example employed is proposition I.32 of Euclid's *Elements*. Before proceeding to the passage, it's worth first noting that between the composition of the *Inquiry* and the first *Critique*, Leibniz's *Nouveaux Essais* had appeared in print.²⁵ There he would have seen the definition of 'intuitive knowledge' as knowledge of necessary truths which arises

when the mind perceived the agreement or disagreement of two ideas immediately by themselves, without the intervention of any other...In this, the mind is at no pains of proving or examining...the truth [As the eye sees light, so] the mind perceives, that white is not black, that a circle is not a triangle, that three [is] one and two. [This] knowledge is the clearest and most certain, that human frailty is capable of. (*New Essays*, Book IV, Ch. ii, §1: 361)

When Leibniz comes to the issue of the nature of intuitive and demonstrative knowledge, one of the examples used is proposition I.32:

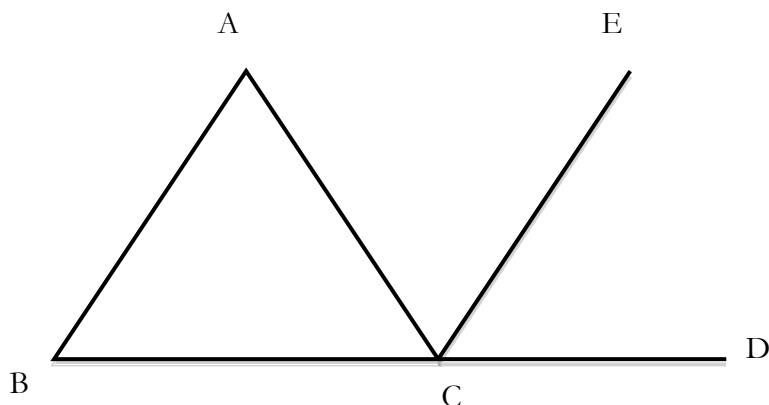
Phil....Now, *demonstrative* knowledge is just a chain of items of intuitive knowledge bearing on ‘all the connections of intermediate ideas’. §2 For frequently the mind cannot join, compare or apply its ideas one to another, and it has to avail itself of one or more intermediate ideas to discover the agreement or disagreement which is sought; and this is what we call *reasoning*. For instance, in demonstrating that the three angles of a triangle are equal to the two right angles, one finds other angles which can be seen to be equal both to the three angles of the triangle and to two right angles. (*New Essays*, Book IV, Ch. ii, §2: 367)²⁶

Demonstrations lack the infallibility of items of intuitive knowledge just because they consist in chains and not in single units of grasping some certain truth (*New Essays*, Book IV, Ch. ii, §7: 368). The example of the *New Essays* suggests one possible origin of Kant’s connection between spatial representation and ‘intuitive’ inference. For Kant, the deployment of diagrammatic representations is not only a required enabling condition for knowledge of necessary truths but the deployment of those representations is also the explanatory basis of the immediate and certain grasp of that knowledge.²⁷

Kant's approach here is similar to the one adopted in the *Inquiry*. He contrasts what a 'philosopher' does, i.e. conceptual analysis, with the practice of the mathematician. The claim is that individual signs are in this case crucial to expressing the relevant necessary truth about triangles:

Give a philosopher the concept of a triangle, and let him try to find out in his way how the sum of its angles must be related to a right angle. He has nothing but the concept of a figure enclosed by three straight lines, and in it the concept of equally many angles. Now he may reflect on this concept as long as he wants, yet he will never produce anything new. He can analyse and make distinct the concept of a straight line, or of an angle, or of the number three, but he will not come upon any other properties that do not already lie in these concepts. But not let the geometer take up this question. He begins at once to construct a triangle. Since he knows that two right angles together are exactly equal to all the adjacent angles that can be drawn at one point on a straight line, he extend one side of his triangle, and obtains two adjacent angles that together are equal to two right ones. Now he divides the external one of these angles by drawing a line parallel to the opposite side of the triangle, and sees that here there arises an external adjacent angle which is equal to an internal one, etc. In such a way, through a chain of inferences that is always guided by intuition, he arrives at a fully illuminating and at the same time general solution of the question. (A716-7/B745).²⁸

This passage is frequently taken by commentators as primarily directed towards showing that the proof-procedure for establishing the truth of proposition I.32 is not performed through conceptual analysis alone.²⁹ In order to perform the task of proving I.32, we must perform the proof outlined in the *Elements*:



Briefly, the proof proceeds as follows: we construct a triangle ABC, then extend BC to point D, and draw a line CE that is parallel to BA. We see that the angle $\angle abc$ is identical to that at $\angle ecd$. Since AC is a transversal of two parallel lines, the opposing angles at $\angle bac$ and $\angle ace$ are equal. We see then that the internal angles of ABC are equal to the sum of the three angles $\angle ecd$, $\angle ace$ and $\angle acb$ (the latter which is held in common). Similarly, we see that those angles are together equal to the sum of two right angles, since we see that those angles together rest upon the straight line BD. Thus the internal angles of ABC must be equal to the sum of two right angles.

The example is not just directed at demonstrating the limitations inherent in any attempt to prove propositions from the mere analysis of the concept *<triangle>* alone. The example is also directed towards showing the *impossibility of acquiring* the full real concept *<triangle>* that we in fact possess through mere analysis of an initial nominal concept. The claim, I'd suggest, is that while *<sum of interior angles necessarily equal to the sum of two right angles>* is an essential part of the content of the concept *<triangle>*, accessing this content could have only come about through a proof procedure detailed. This alternative reading suggests that Kant's claim is that this content cannot be acquired through mere analysis of the bare nominal concept of a triangle alone. Support for this

reading can be found from Kant's metaphysics lectures. There Kant discusses the same example, and characterizes it as one in which we are to think of both the philosopher and geometer beginning with a thin understanding of *<triangle>* and attempting to form a fuller version of that concept:

Although synthetic judgments can be *a posteriori* judgments, they can also be judgments cognized *a priori*. But even *a priori* reason must add something that did not lie in the concept....

...in general synthetic judgments are possible only by means of the corresponding intuition and the concept formed and added to this, and that this intuition must happen *a priori* if the judgment is to be posited *a priori*.

E.g., the three angles of the triangle are equal to two right angles of 180° , is a synthetic judgment: for this proposition cannot be brought out of the analysis of a figure enclosed by three lines, rather it must be made at least in thought experiments for finding it and through that the proof is thought. (*Metaphysik Vigilantius* (K3), 29:968-9)

Kant evidently takes the metaphysician and geometer to be operating with no more than the *nominal* concept *<triangle>*, captured just as *<figure enclosed by three straight lines>*, i.e. the initial understanding of the concept that Euclid afforded himself in order to begin his proof procedure. Part of what we are supposed to grasp from the discussion of the I.32 example is that there is no available inferential transition from the concept *<figure enclosed by three straight lines>* to *<internal angles necessarily equal to the sum of two right angles>* just through a piece of analysis of the former concept. The latter propositional content cannot be justified just because the latter propositional content cannot be 'brought out of the former content. Why this is so is because there are steps in the diagrammatic proof that, although properly characterised as achievements of understanding, are not

properly characterized as *inferences* in the common Early Modern sense at all, i.e. as steps that might be reconstructed in a piece of syllogistic reasoning with an intermediate middle term. At vital points in the proof of I.32, the steps are not inferred in syllogistic reasoning but are just represented diagrammatically.

The counterfactual claim employed in the thought experiment is one whereby we are to see that, were we (say) the kind of agents Hume characterizes us as being, we would in that case never have acquired the mathematical version of the concept <triangle> that we in fact possess. The conceptual content whose origin Kant is seeking to explain is that of <necessity>, but the only way we could have acquired *that* conceptual content is if somehow necessity was manifested within the phenomenology of our perceptual or imaginal experience. Just as a matter of fact, both Hume and Kant agree, sensory content doesn't carry this modal inflection within it. But although we can't *sense* that something must be the case, we do in fact *perceive* that something must be the case, as when performing Euclidean proofs. This achievement only occurs, Kant thinks, through the use of individual signs in the performance of inferences that themselves provide the possession condition of concepts.

This allows for an understanding of the original import of Kant's use of the example of the 'first geometer' in the B-preface to the *Critique* (Bxi-xii). There Kant lauded the first geometer for recognizing that the nature of the task at hand involved neither 'reading off' properties from a drawn figure, nor reading off from a concept, but instead drawing out from the proof procedure what the geometer had himself put in. The procedure Kant outlines here with regard to proposition I.32 is not that of just reading off properties from pictured *instances* of the initial concept <triangle> – all that would do would be to present a series of images of three straight lines laid end-to-end. What Kant is envisaging here is the concept <triangle> obtained via a caricatured empiricist concept acquisition process. Were we agents that acquired our mathematical

concepts by way of an empiricist model, through a Humean copying or Lockean abstracting from instances of presented sensory particulars, the concepts formed would only contain the content that reflects what we had read off from those presented particulars. But in that case the concept acquired would not include the content *<necessity>* at all, since the contents acquired would only have been perceptually presented as contingent (even if uniformly observed) properties of the representations of triangles (A718/B746). Yet, Kant claims we come to possess a concept that includes the content that the sum of the internal angles *necessarily* equals 180° degrees as part of the essential intensional content of that concept.

For Kant, the task is to explain the modal phenomenology of geometrical judgment, i.e. how it is that we could have come to make judgments that are ‘combined with consciousness of their necessity’ (B41). I could of course gain some epistemic confidence that all triangles’ internal angles equal 180° inductively, but the judgment that they necessarily do so is a judgment with a distinct intension. Repeated perceptions that I have two hands might make it psychologically compelling for me to assent to that proposition, but no amount of such repetitions would engender in me the distinct judgment that it could not be otherwise. Yet, Kant holds, a single performance of the proof procedure for I.32 can make this distinct component of the judgment cognitively available to me, just because it is a procedure that allows me to perceive that it must be the case. Without such a procedure, we would not grasp the proposition’s truth in the way that we in fact grasp it.

The counterfactual reasoning is supposed to hold equally well against Leibnizian rationalism represented by Wolff and Mendelssohn.³⁰ In his winning submission to the Prize Essay competition Mendelssohn maintains that we are able to untangle or unpack the concept *<extension>* into all the truths of geometry.³¹ Kant’s challenge is simply to untangle the conceptual content *<sum of interior angles necessarily equal to the sum of two right*

angles> from the initial concept <*figure enclosed by three straight lines*>. Decompose that initial concept all you like, Kant thinks, and the former constituent just won't reveal itself. Analyzing <*figure enclosed by three straight lines*> reveals nothing more than the original constituents: <*figure*>, <*three*>, <*straight*>, <*line*>, etc. The situation for the rationalist is the same it is for the empiricist – were our concept acquisition procedure the one envisaged, we would never have acquired the mathematical concept <*triangle*> that we in fact possess. Moreover, without those possessed concepts we would never be able to express the propositions that we can in fact express. Neither of these approaches can explain then how these synthetic a priori judgments are possible.

5. Construction and Generality

My claim is not that every occurrence of construction is always an action of concept acquisition, since surely Kant thinks that one may repeat the same proof without thereby acquiring the concept anew. Rather, my claim is that the conditions under which a geometrical concept is 'originally given' are just those construction procedures. In his reply to Kästner, Kant seems to acknowledge this point:

However, that the possibility of a straight line and a circle can be proved, not mediately through proofs, but only immediately, through the construction of these concepts (which is not, to be sure, empirical), stems from the fact that among all constructions . . . some must be the first.³²

My account might seem vulnerable to the objection that in Kant's own presentation of an example of mathematical construction, that of the I.32 proof procedure, he clearly

sets up the case whereby each inquirer, the philosopher and the geometer are *given* the concept triangle before the construction procedure can take place. If this is correct, then of course the procedure itself cannot constitute the conditions for possession of the concept *<triangle>*. However, there are reasons already seen that lead one to think that this is not the case. Firstly, the passage quoted earlier from the *Metaphysik* *Vigilantius* shows Kant envisaging the case as one whereby the inquirer possesses the content *<figure enclosed by three straight lines>* but does not already possess the content *<internal angles necessarily equal to the sum of two right angles>* and where the goal is to form a new concept including this latter content. Secondly, it has also been already shown that in the case of empirical concepts, Kant allows that we can have some initial grasp of the content without our thereby possessing the definition of the concept. Kant makes reference to the notion of a ‘putative definition’ or ‘**designation**’ which he claims is only the elaboration of a minimal set of marks that allow one to identify a subject matter so that one can subsequently conduct experiments on that subject matter, thereby securing some knowledge of the essential marks of the kind under consideration (A728). There is no reason not to think that in the construction example, Kant is similarly imagining two inquirers with the nominal or putative definition of *<triangle>*, one which does not sufficiently count as the definition proper. Thirdly, Kant’s analysis of definition is premised on the idea that there is an ordinary or common signification attached to the natural language expression of a conceptual content that is prior to the proper definition. As was seen earlier, in the *Inquiry* Kant uses the distinction between the ‘ordinary signification’ and the mathematical definition of the concept *<cone>*:

The concept which I am defining is not given prior to the definition itself, on the contrary, it only comes into existence as a result of that definition. Whatever the concept of a cone may ordinarily signify, in mathematics the concept is the product

of the arbitrary representation of a right-angled triangle which is rotated on one of its sides. (*Inquiry*, 2: 276).

Here Kant gives an example of a concept of which we might have an ordinary grasp, perhaps based on a simple perceptual individuation capacity, and which gives us an initial intension. However, it does not constitute a grasp of the definition of that concept. The definition of *<cone>* is only grasped coterminously with the production of a cone through a representational operation upon an intuition of a triangle, since it is the case both that the definition provides the method for first representing a token of that type and that the representation of a token of that type is required to express the operational rule that forms the definition. Fourthly, the example at I.32 is understandable as based on a subject grasping a concept but without a clear grasp of the genuine intension that characterizes the essential marks of its extension. But this is not an unusual cognitive state within Kant's theory of knowledge, since this is a cognitive state that familiarly occurs in any grasp of an analytic judgment. In making an analytic judgment, such as that 'all bodies are extended', I am predicating of the subject concept a property that is in fact 'contained' within that subject concept. For such judgments to be possible in a way that presents the illusion of being informative, Kant allows that we can think the predicate concept 'confusedly' in the subject concept (A7/B11). Yet Kant would not deny that such a confused subject does not also have some minimal grasp of the intension of *<body>* in such a case. The notion between there being a graspable but only partially articulated conceptual content is implied by Kant's very notion of analytic judgment. That same state is the one that is attributable to the mathematician prior to the performance of the I.32 proof.

This leads to a second objection to this reading, namely that it threatens to render the relevant propositions *analytic* rather than synthetic a priori, since on my account the

propositions in question are capable of being cognised just off the basis of gaining a proper understanding of the relevant concept in the subject position of the judgment. However, it doesn't follow that the judgement in such cases is analytic. Firstly, Kant frequently speaks of synthetic knowledge being in one sense knowledge that is made on the basis of some concept possession:

If one is to judge synthetically about a concept, then one must go beyond this concept, and indeed go to the intuition in which it is given. For if one were to remain with that which is contained in the concept, then the judgment would be merely analytic, an explanation of what is actually contained in the thought. (A721/B749)

Kant here characterizes judging synthetically as judging *about* a concept. He characterises synthetic a priori judgments as ones that identify features that 'belong' to the concept even though they do not 'lie in it' (A718/B746). In the *Transcendental Analytic*, Kant says that as soon as sensible conditions are involved, 'synthetic judgments that flow *a priori* from pure concepts of the understanding' can be determined (A136/B175). In the *Transcendental Aesthetic*, Kant similarly claims that there are synthetic a priori cognitions that 'actually flow from the given concept' of Space (B40). At A9/B13, Kant characterizes synthetic judgment in relation to causal judgment whereby 'the understanding 'seeks 'to discover beyond the concept of *A* a predicate that is foreign to it and which it nevertheless believes to be connected with it'. If we take Kant at his word here then there is nothing contradictory in the claim that in making a synthetic judgment we are making claims that are in some sense true in virtue of the nature of the concept that features in the subject position of the judgment. The very distinction between analytic and synthetic judgment is not drawn in terms of whether a predicate is connected with a concept in the subject position or not; both types of judgment are

based on such a connection. The distinguishing factor is whether the relation of the connection between the concepts is based on containment or some other determining ground.

It cannot be therefore that a judgment is analytic if and only if it is made on the basis of concept possession alone. It is rather that we are making an analytic judgment about that concept on the basis of our possession of that concept by appealing to the intensional features ‘contained’ in that concept, in Kant’s particular notion of containment.³³ This is obscure in itself, and obviously more needs to be said here, not least with regard to the parameters determining Kantian analyticity. Why, for example, would an acquired mathematical concept of *<triangle>* not become the *new* initial version for our subsequent inquiries? Clearly it does not, since Kant held that what we can analytically infer from the concept *<triangle>* does not alter as a result of the construction procedure. Yet the thought that it might do so only stems from the thought that somehow the parameters as to what contents are ‘contained’ in the concept *<triangle>* might alter as a result of the construction procedure, and there is no reason to think that he endorses this latter claim.

Although the construction procedure would be a process enabling possession of a concept and the generation of knowledge in virtue of that possession, it does not follow that the knowledge generated is analytic. Rather construction is a concept-acquisition procedure that involves the acquisition of essential marks of the concept that belong to it and yet are not contained in it. The procedure *can* reveal those non-contained essential intensional marks, but only by virtue of deploying some non-conceptual representational capacities and putting the relevant particular intuitions to use in inferential processes. The resulting judgments will be a priori just because the concept acquired expresses necessary truths about the extension and has occurred via the

provision of an a priori intuition; they will be synthetic because the concept acquired has been acquired through the mediating domain of sensible intuition.

Kant expresses in the *Critique* the confidence he showed in the *Inquiry* that in mathematical cognition we can think ‘the universal in the particular.’ (A714/B742) Yet it is clear that by this time Kant was not insensitive to the generality problem. In the Schematism section, Kant in fact cites the construction procedure performed on intuitions as the grounds for resolving this problem. There he claims that there are schematic rules for the construction of triangles that allow us to avoid the dependence on particular ‘images’ of triangles:

In fact it is not images of objects but schemata that ground our pure sensible concepts. No image of a triangle would ever be adequate to the concept of it. For it would not attain the generality of the concept, which makes this valid for all triangles, right or acute, etc., but would always be limited to one part of this sphere. The schema of the triangle can never exist anywhere except in thought, and signifies a rule of the synthesis of the imagination with regard to pure shapes in space. (A140-1/B180)³⁴

Whether we construct a triangle empirically on paper or imaginatively in our mind’s eye, there will always be some sensory image employed, either directly or indirectly within our visual imagination. It is the intuitional content imported into any sensory image deployed though that accounts for how the produced individual sign can express general truths about the extension of the concept. It is not the sensory imagistic content of the sign in concreto that is being attended to when we reason with triangle-images but rather the intuitional content contributed to and entangled within those representations.

This content is only accessed through the construction procedure. One might ask

why this should resolve the generality problem though, rather than moving the problematic bump in the rug from the particularity of sensory content to the particularity of intuitional content. Why should the mere appeal to acts of construction grant us the security we need to infer general truths about the class? Kant's answer to these questions must involve his commitment to the claim that 'that which follows from the general conditions of the construction must also hold generally of the object of the constructed concept' (A716/B744). This though just raises the same question anew, plus another. What determines that the conditions of construction are themselves general? Furthermore, on what grounds can one infer from the general nature of the concepts to the general nature of the objects that fall under them?

Addressing these questions is beyond the scope of the present paper.³⁵ This reading, if correct, does however suggest three points of general relevance to the understanding of Kant's Critical Philosophy, points that I will merely raise here. The first is, as previously mentioned, that it is an oversimplification of Kant's methodology if one attributes to him the view that the acquisition conditions of a concept are irrelevant to the correctness conditions for its use. At least with regard to the example of geometrical concepts, this is not the case. On the contrary, the correctness conditions for such concepts' use are given by the construction conditions for those concepts: a concept applies to the extension that is exemplified on occasions of that concept's construction. It is unclear how we are to understand Kant's general normative turn in epistemology however if it does not imply a critique of the investigation into concept acquisition conditions as a general methodology.

Secondly, there is the related point regarding Kant's inquiry being one into the origin of the concept *<necessity>*. Kant's claim is that this concept serves as a constituent content of both categorial and pure sensible concepts. Yet it is clear, I would claim, that his account of the justification of the use of this concept with regard to geometrical

concepts is just to show how the relevant concept is originally formed. This suggests that Kant's general response to Hume should be understood not as limited to the justification of the concept *<cause>* but rather of the conceptual constituent conceptual content *<necessity>*. Furthermore, it suggests that his strategy for justifying that latter concept is identical to the task of explaining its valid acquisition.

Finally, there is the issue with regard to the 'sense and significance' of a priori concepts. Kant appears on occasion to hold that categorial concepts might have an explanatorily prior and purely *logical* content that is capable of being first articulated in isolation from the conditions of sensibility, but that they subsequently receive further 'sense and significance' with their application in the context of possible experience (e.g. A54/B78, A147/B188, A219/B266-7, A239/B298). However, Kant's approach with regard to the a priori concepts of geometry involves an identification of the original conditions of their acquisition not with a prior abstract logical formulation of their content in purely discursive terms, but rather with their original generation as rules concretely manifested within spatiotemporal representation. The question raised concerns how we are to understand the explanatory priority relation between abstract and concrete expressions of discursive content for a priori concepts in general. My aim here has been to defend that thought that Kant does in fact think that uncovering the general conditions of the acquisition of geometrical concepts can suffice to legitimate those concepts in use. They do so by involving the representations of pure intuition as the fundamental referents of our geometrical knowledge claims. Ultimately then geometrical concepts are the literal products of our reflections upon the most general features of outer sense.³⁶

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¹ E.g. A95, A129, B167, A310/B366, A 720/B748. Kant uses 'a priori' to modify a number of terms, such as 'intuition', 'representation', 'principle' 'judgment', 'truth', etc. I don't explore the relationship between this and other uses of the modifier here.

² I use the angled brackets and italics to indicate mention of the concept. I don't discuss here the concepts <Space> and <Time>, which also fall under the heading of pure sensible concepts, and which have their own acquisition procedure, as they relate to the pure intuitions of Space and Time.

³ E.g., *On a Discovery*, 8:221. See also *Inaugural Dissertation*, §8, 2:395, §15, 2:406; *Metaphysik Moronovious*, 29: 760–3.

⁴ E.g., see the *Blomberg Logic*, §254; *Jäsche Logic*, §3.

⁵ At A729/B757, having claimed that mathematics proceeds through the particular process of construction, Kant states that only mathematical concepts are apt for this process. As shall be discussed, only mathematics contains *definitions*, Kant thinks, because only mathematical concepts can be acquired through being defined in a construction procedure. Kant does speak of the acquisition procedure for categorial concepts as being of a specific kind, and he refers to this procedure as 'original acquisition' (*On a discovery*, 8:221). That a priori concepts are acquired at all might seem surprising, since whatever else it connotes, 'a priori' surely connotes some sort of 'independence from experience'. One can point out first of all that for Kant, a concept is a priori if and only if it issues in an a priori *judgment* when deployed, where the latter is understood as one whose truth conditions are not provided by sensory experience (even if sensory experience nevertheless serves as a necessary enabling condition for the possibility of such judgments – B1–2). This condition does not by itself place any putative restrictions on a priori concepts' possession conditions. Nevertheless, the very idea of categorial concepts as acquired might seem to be precluded by the fact that those concepts are for Kant necessary conditions of the possibility of experience – for discussion see my [2011]).

⁶ I focus on the case of geometry in this paper. Although there are of course notable and important differences between Kant's handling of geometry and his handling of arithmetic and algebra, though there are grounds for thinking that geometrical knowledge was for Kant the paradigmatic case of mathematical knowledge (his mathematical examples are primarily Euclidean examples (e.g. Bxi–xii, A164/B205, A716–7/B745).

⁷ There are well-known complications here with regard to interpreting the nature of the ‘synthetic’ and ‘analytic’ methods supposedly deployed in the *Critique* and the *Prolegomena* respectively, whereby only the latter is supposed to be pursued upon the assumption of some well-grounded body of knowledge. However, I take it to be well established that Kant frequently argues from the basis of the assumption of some a priori knowledge in the form of mathematical knowledge (most notably in the argument for transcendental idealism in the Transcendental Aesthetic) – see Bx, A4/B8, B4, B20, A38-9/B55-6. For a recent discussion of the meaning of Kant’s synthetic method, see Merritt (2006).

⁸ That Kant is interested in securing epistemological (and not merely psychological) results does not entail that his inquiry cannot be construed as one into the literal origins or sources of our concepts, although it does require re-consideration of the type of epistemic normativity that might be at stake here, as I argue in my (2011). I take it that the account presented here is in some ways supportive of the picture presented in Longuenesse (1998).

⁹ Henceforth, the ‘*Inquiry*’.

¹⁰ Kant’s insistence here on looking to actual practice in order to determine proper method is repeated in the *Inaugural Dissertation*, where he states that ‘in natural science and mathematics, *use gives the method*’ (§23, 2:410 – emphasis in original).

¹¹ The essential character of mathematical concepts as ‘elective’ is noted by Sutherland (2010).

¹² See *Jäsche Logic*, §§4-5.

¹³ However, caution is required here, since Kant is not explicit with regard to what in fact determines the parameters for givenness in this sense.

¹⁴ As Kant puts it:

In mathematics I begin with the definition of my object, for example, of triangle, or a circle, or whatever. In metaphysics I never begin with a definition. Far from being the first thing I know about the object, it is nearly always the last thing I come to know. In mathematics, namely, I have no concept of my object at all until it is furnished by the definition. In metaphysics I have a concept which is already given to me, although it is a confused one. My task is to search for the distinct, complete and determinate concept. (*Inquiry*, 2:283)

This is a claim that Kant retains in the first *Critique*. There he holds that, it is still the case that mathematics begins with definitions, although it is unclear as to whether or not he now holds the achievement of definition in metaphysics to be possible at all. (A729-32/B757-760).

¹⁵ It is clear then that in 1763 Kant still viewed the proper method of metaphysics to be that of analysis, a view famously rejected in the *Critique*.

¹⁶ This distinction corresponds to types of representational vehicle. Both types of representation can serve to express general content – e.g. <triangularity> can be expressed through a picture of a triangle or through the tokening of the word ‘triangle’ – the difference consists in the manner in which each type of representational vehicle expresses that same representational content. When considered with regard to our performance of mathematical operations, the distinction broadly corresponds to the contemporary one in developmental psychology and neuroscience between *nonsymbolic* and *symbolic* numerical cognition, i.e. the respective products of our abilities to represent quantities through dots, strokes, etc. on the one hand and to symbolize those representations in terms of Arabic or Roman numerals or natural language on the other, e.g. see Ansari, Chee and Venkatramana (2005), Fias and Verguts (2004), Lipton and Spelke (2005), Spelke (2011). Parsons (1983) gives an illuminating discussion of the possible role of ‘concrete tokens’ deployed for the purpose of ‘verifying general propositions’ (136). Though my emphasis on concept acquisition of course differs significantly from the approach pursued there, my account of the epistemic role of signs *in concreto* (which is performed by *intuitions* in the *Critique*) is broadly in accord with Parsons’ account of intuition. For differing accounts, see Hintikka (1969), Howell (1973) and Thompson (1972).

¹⁷ See Friedman (1985), Shabel (2010), Young (1984).

¹⁸ I discuss this theme more in my ‘Kant on Signs *in Concreto* in Geometry’ (*ms*).

¹⁹ The notion of intuition itself was made in the *Inaugural Dissertation* – the recognition of its necessary co-deployment with concepts for cognition, i.e. the Discursivity Thesis, was not made until the first *Critique*.

²⁰ This point is stressed in Warren (1998) and Waxman (2005).

²¹ Either individually or coterminously, as when we can access spatial information through both touch and sight.

²² Although Kant is not clear on this point, I see no reason to take him as claiming that our imaginational access to intuitional content does not involve sensory content, but rather that it involves an indirect reproduction of such content.

²³ When Kant is discussing the importance of the distinction of mathematics and philosophy, he notes that mathematicians have rarely philosophized regarding the nature of their own practice. He chastises them for neglecting that task, which he then characterizes as that of accounting ‘[f]rom whence the concepts of space and time with which they busy themselves...might have been derived’ (A725/B753).

²⁴ Typical statements of the dangers of invented concepts can be found at A222-B279.

²⁵ Cassirer (1983) claims that Kant read the *Nouveaux Essais* sometime between its publication in 1765 and the writing of the *Inaugural Dissertation* in 1770 (97-99), though he offers little justification for the claim. Tonelli (1974) adduces evidence for thinking that any familiarity Kant

had with the work could not have occurred second-hand through its reception by his contemporaries. I think a case can be made for Kant's first-hand familiarity with the *Nonveaux Essais* (e.g. he refers to the 'Essays of Locke and Leibniz' (4: 257) in the *Prolegomena* two years after the publication of the first edition of the first *Critique*) though to do so would be beyond the scope of this paper. In what follows I present one example of the similar themes and modes of expression to be found in both the *Nonveaux Essais* and in Kant's Critical writings.

²⁶ The speaker here is Philalethes, who is Locke's representative, though Leibniz does not have Theophilus – his representative – quarrel on these points. The conception of inference expressed by Locke would have been a common one within Cartesian and Port-Royal Logic. Hume too would have subscribed to it, challenging not the conception of inference at stake, but rather the scope of the knowledge that might be attained through it. For an excellent discussion of these topics see Owen (1999).

²⁷ The passage is worth considering not least since it gives one likely contender for the source of Kant's focus upon the word 'intuition' (*Anschauung*, but which Kant also refers to with the Latin *intuitus*) that connects it with the epistemic sense of intuitive knowledge found in the rationalist tradition.

²⁸ Whereas Leibniz describes the demonstrative reasoning employed in proving Proposition I.32 as a 'chain of items of intuitive knowledge [*enchaînement des connaissances intuitives*]' Kant's reasoning with regard to the same proposition is held to proceed through a 'chain of inferences [*eine Kette von Schlüssen*]' that is always guided by intuition'.

²⁹ E.g. Friedman (1985), Shabel (2003), (2006).

³⁰ I have presented the critique here as if an empiricist account of geometrical concept acquisition is Kant's target. However, I argue in my (2014) that the primary target of the critique is in fact Mendelssohn's rationalist approach. Both positions are, I would claim, effectively criticised in the passage. Shabel (2004) argues that one of the targets here is one who employs empirical methods of proofs with regard to Proposition I.32 and that this target was in fact Wolff, who presented an account whereby the geometer proceeded with particular claims regarding the management of the compass, etc. (209-212). Perhaps Kant does have Wolff's proof in mind here, as it would give a plausible alternative of what it would be to 'read off' properties of a figure. Similarly, Dunlop's (2013) account of Wolff's theory of geometrical concept acquisition might suggest that he is the target here, since that account seems to imply the adequacy of the acquisition of the concept <triangle> from occasions of perception of triangle instances (462).

³¹ See my (2014) for discussion.

³² 'Über Kästner's Abhandlungen,' (20: 411), translated by. D.R. Lachterman, quoted from Lachterman (1989, 53).

³³ For discussion of Kant's notion of analyticity and containment see e.g. Anderson (2004) and (2005), de Jong (1995) and Proops (2005).

³⁴ This passage is traditionally thought to be indicative of Kant's familiarity with Berkeley's criticism of Lockean abstract ideas in the *Principles* (e.g. [Guyer 1998, 165]).

³⁵ I have not attempted to give anything like a complete account of the relationship between geometrical concepts, geometrical schemata, and spatiotemporal intuition. Nor have I attempted to adjudicate here with regard to how this account might figure within recent debates in Kant's philosophy of mathematics. However, it is perhaps worth noting some potential relevance in regard to one such recent debates, that between Michael Friedman, and Lisa Shabel concerning the status of diagrammatic reasoning in Kant's philosophy of geometry. One of the points of concern is how generality might be expressed via a particular image contained in a diagram. Friedman's claim is that it is clear that for Kant the generality is contributed by virtue of the conceptual representations the subject possesses prior to the construction procedure involving particular diagrams:

In particular, whereas such diagrammatic accounts of the generality of geometrical propositions, as we have seen, begin with particular concrete diagrams and then endeavor to explain how we can abstract from their irrelevant particular features (specific lengths of sides and angles, say) by relying only on their co-exact features, Kant begins with general concepts as conceived within the Leibnizean (logical) tradition and then shows how to "schematize" them sensibly by means of an intellectual act or function of the pure productive imagination. (Friedman, 2012, 239)

On the interpretation suggested here, Kant's rejection of the Leibnizean logical tradition is more thoroughgoing than Friedman envisages. This is so, I claim, since for Kant the concept is not possessed prior to the schematization process. Rather we acquire the explicit discursive representation in the course of schematizing over intuition. This is the sense in which the concepts are 'originally acquired'. My reading thus supports Manders's (2008) account of 'conceptualization via the diagram construction conditions' (74).

³⁶ For comments on earlier versions of this paper I am grateful to audiences at Humboldt University, University of Amsterdam, University of California at Berkeley and Clare College, Cambridge

Dear Emily,

I've attempted to improve the clarity of the piece by making changes to each of your comments on the following pages of the previously submitted document:

- p. 5 – the nature of the acquisition of given concepts
- pp. 6-7 – the meaning of rule-governed operations in this context
- p. 8 – the meaning of inference in this context
- p. 10 – clarity on the passage regarding imaginational use of pure intuition.
- p. 18 – reformulation of sentence on the influence of Leibniz.

I've also tried to catch the various remaining typos and word substitution suggestions.